

# **Numerical Atmospheric Modeling**

**Song-You Hong**

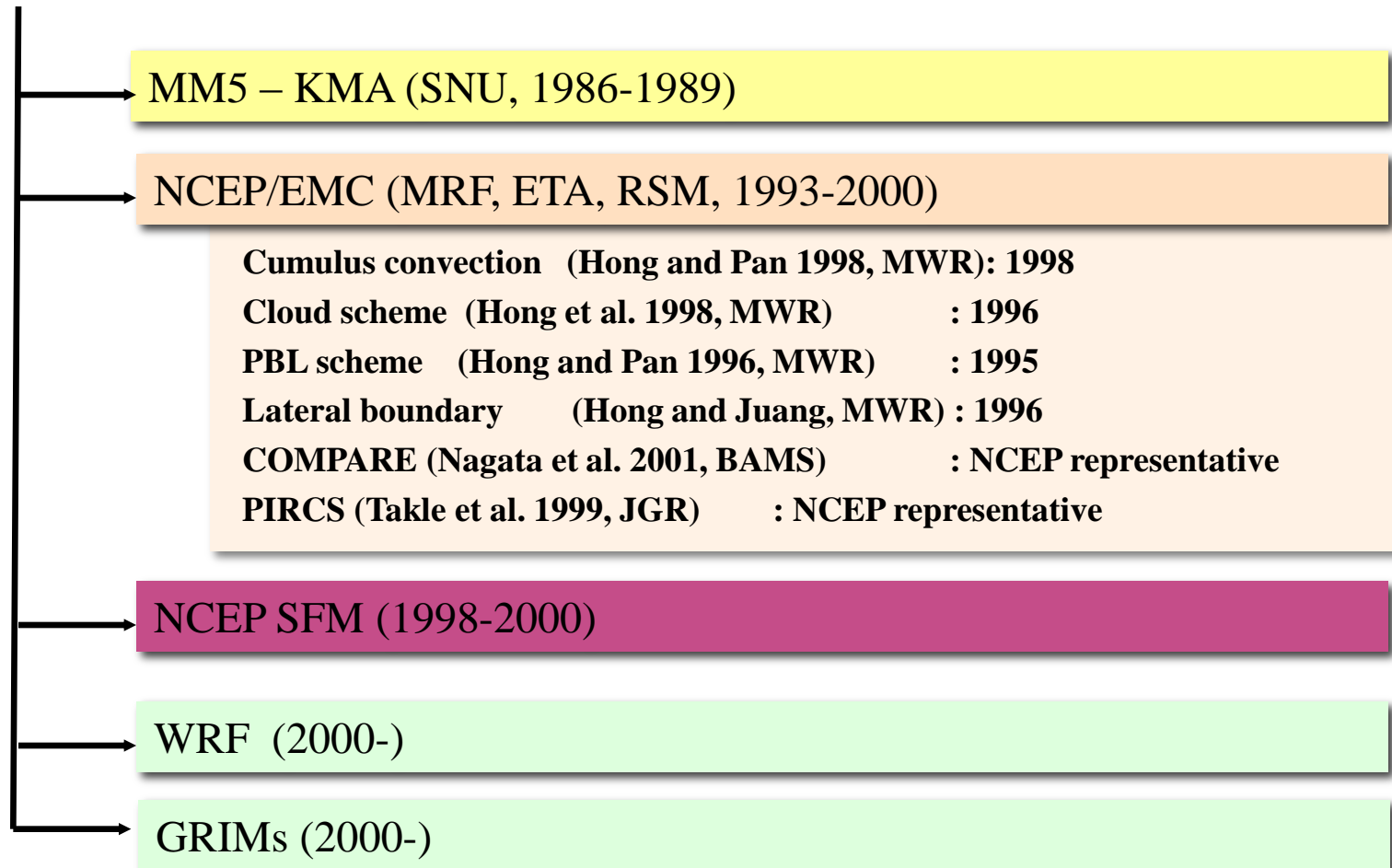
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# List of presentations

- **Concept of modeling**
- **Structure of models**
- **WRF model**

# Modeling experience

**Papers : about 100**

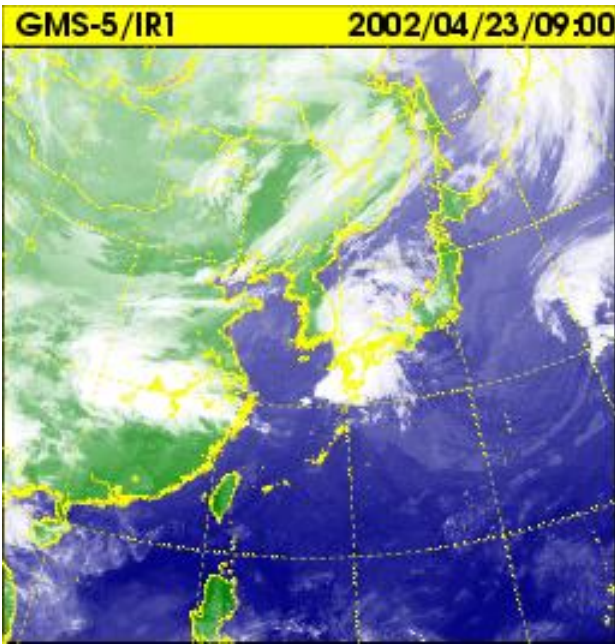


How the today's forecasts were made ?

Observation



Forecasts



NO!

광역예보 [ 충청북도 ]							
23(화)	24(수)	25(목)	26(금)	27(토)	28(일)	29(월)	30(화)
CSO W365.COM		오늘 23(화)		내일 24(수)		모레 25(목)	
	오전	오후	오전	오후	오전	오후	
최저/최고기온(°C)	(13)	23	5	22	3	23	
강수확률(%)							
풍향	SW-NW		NW-NE		NW-NE		



Then, what ?

Weather forecasts ....

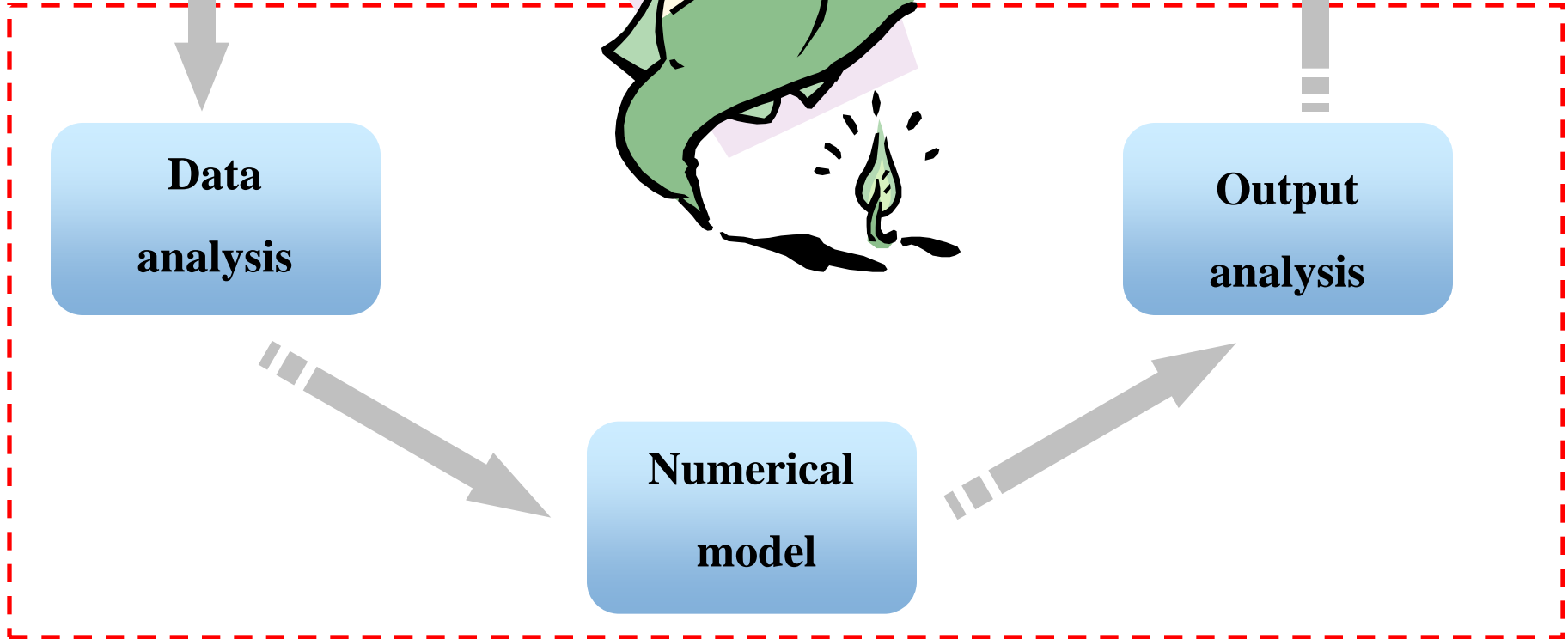
Observation

Forecasts

Data  
analysis

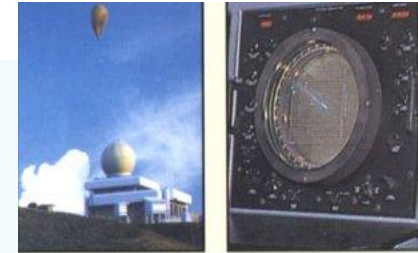
Output  
analysis

Numerical  
model

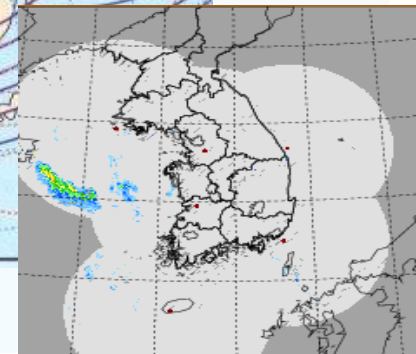
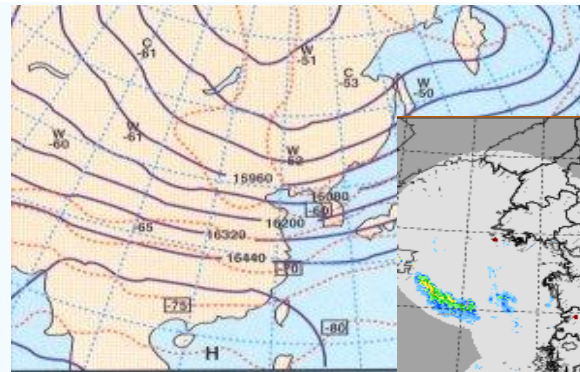




Step1: Observation



Step2: Data analysis



# Theory ?

## Thermodynamics

Heat = Energy + Work

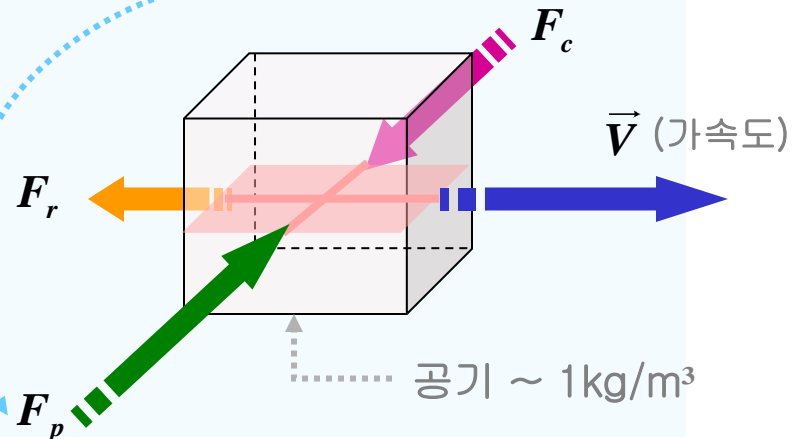


$$\begin{aligned}\Delta H &= c_p \Delta T - \alpha \Delta p \\ &= c_v \Delta T + p \Delta \alpha\end{aligned}$$

## Dynamics

Force = Mass × Acceleration

- Mass  $\approx 1 \text{ kg/m}^3$
- Force: **PGF, CO, Friction...**



- Momentum

$$F = ma$$

- Mass

$$\frac{1}{M} \frac{dM}{dt} = 0$$

- Moisture

$$\frac{dq}{dt} = E - C$$

- Ideal gas

$$p\alpha = RT$$

- Energy

$$Q = C_v \frac{dT}{dt} + p \frac{d\alpha}{dt}$$

# CONSERVATION

## The governing equations

*V. Bjerknes (1904) pointed out for the first time that there is a complete set of 7 equations with 7 unknowns that governs the evolution of the atmosphere:*

$$\frac{d\mathbf{v}}{dt} = -\alpha\nabla p - \nabla\phi + \mathbf{F} - 2\boldsymbol{\Omega} \times \mathbf{v} \quad (1-3)$$

$$\frac{\partial\rho}{\partial t} = -\nabla \cdot (\rho\mathbf{v}) \quad (4)$$

$$p = \rho RT \quad (5)$$

$$\frac{ds}{dt} = C_p \frac{1}{\theta} \frac{d\theta}{dt} = \frac{Q}{T} \quad (6)$$

$$\frac{dq}{dt} = E - C \quad (7)$$

7 equations, 7 unknown (u,v,w,T, p, den and q)

solvable

## History of numerical weather forecasts

1904 : Norwegian V. [Bjerknes](#) (1862–1951) :  
Setup the governing equations

1922 : British L. F. [Richardson](#) (1881–1953) :  
Integrate model → failed

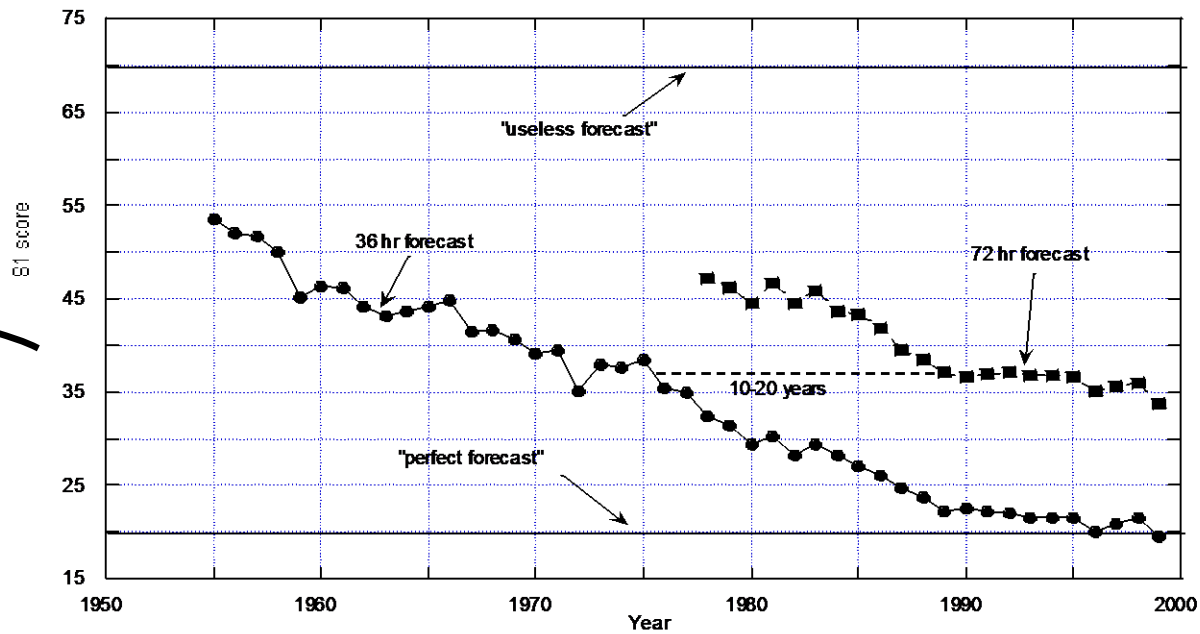
1939 : Swedish C.-G. Rossby :

1948, 1949, J. G. Charney (1917–1981)

1950 : Princeton Group ([Charney](#), [Fjortoft](#), [von Neuman](#))  
[ENIAC](#) (Electrical Numerical Integrator and Computer)  
→ first success

# Tendency of forecast error (1955-1998) : NCEP

NCEP operational S1 scores at 36 and 72 hr over North America (500 hPa)

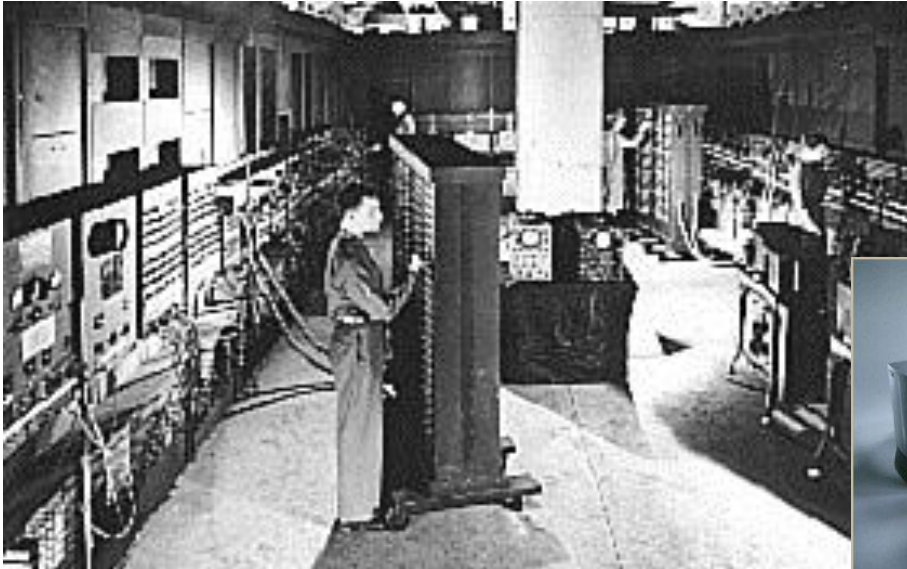


1day / 8 yrs

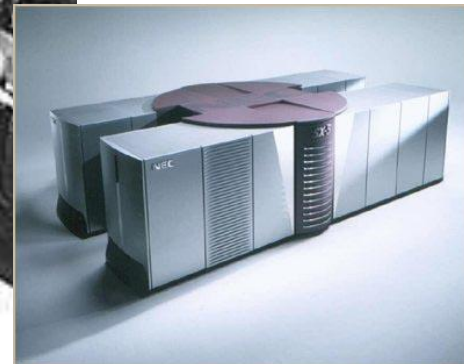
# Factors for the improvement

- supercomputers
- physical processes
- initial conditions

# Super-computer for weather models



ENIAC, 1946



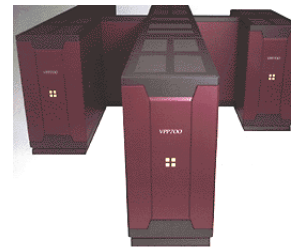
NEC SX-5



Cray T3E



Cray SV1



Fujitsu VPP700E

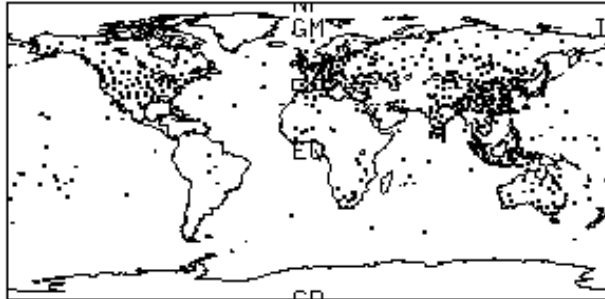


Cray T90

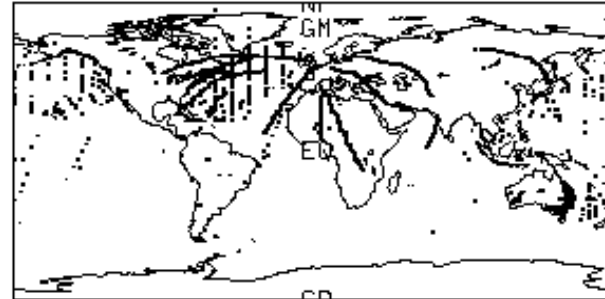
**Initial condition  
(data assimilation)**

# Various observations

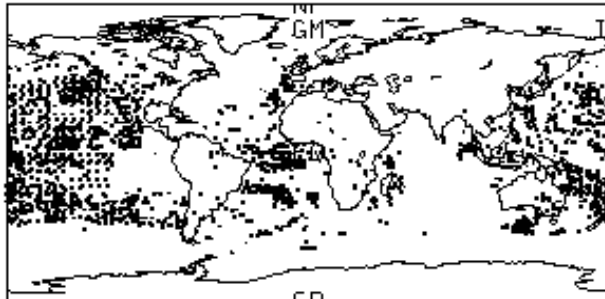
RAOBS



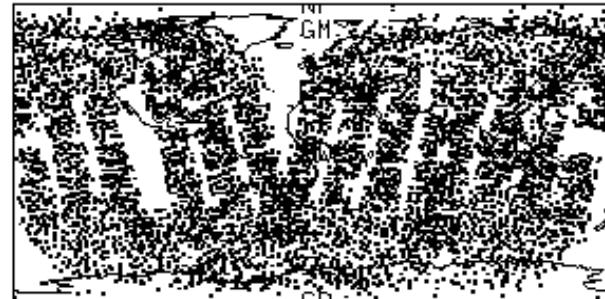
AIRCRAFT



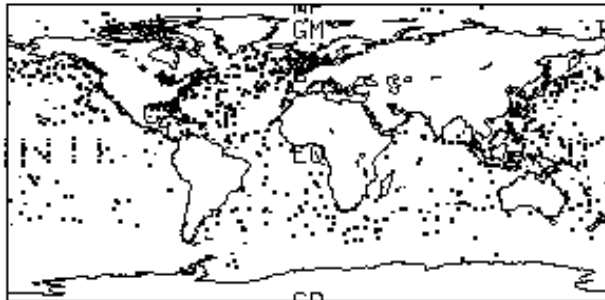
SAT WIND



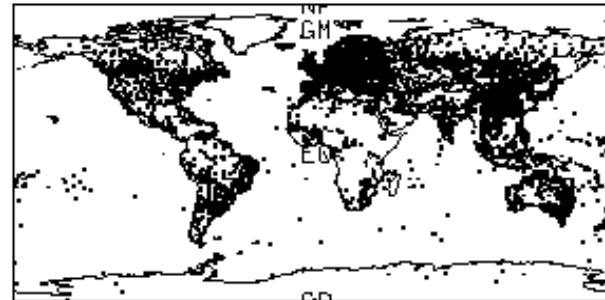
SAT TEMP



SFC SHIP



SFC LAND



# Data Assimilation

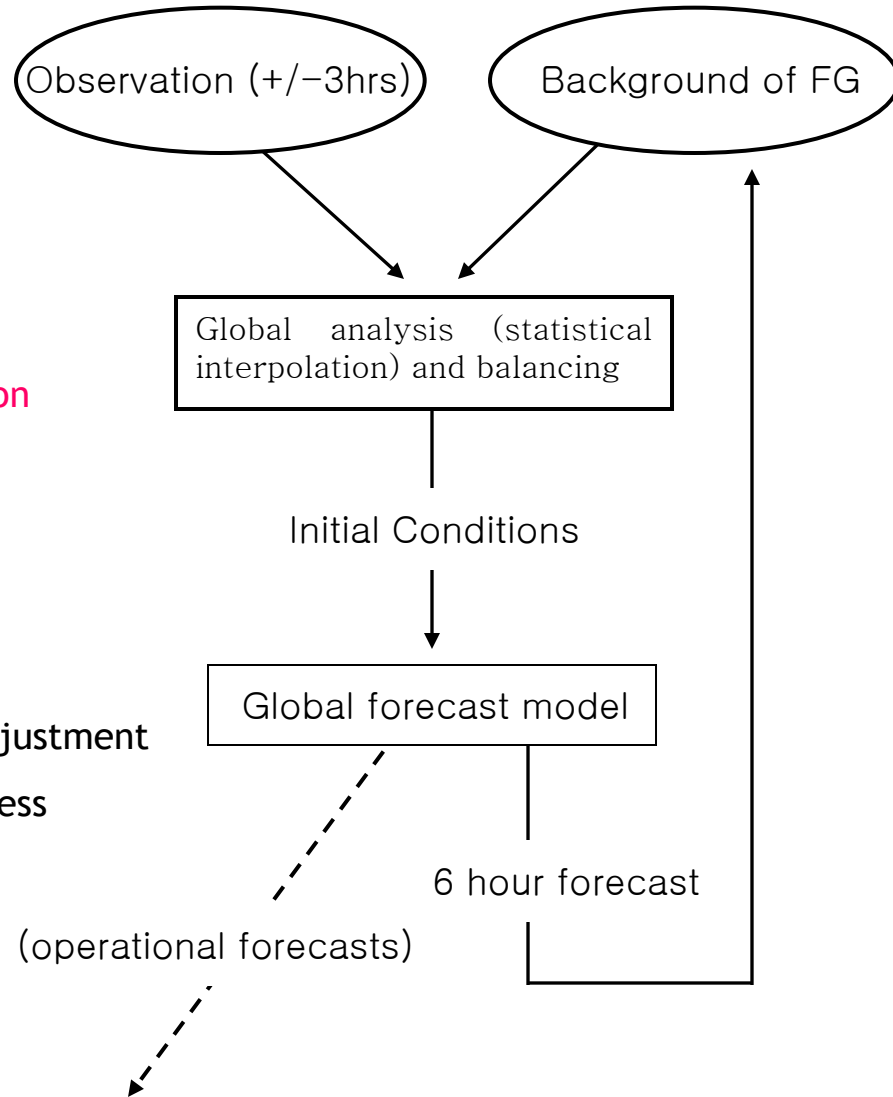
- Model 1°X 1° resolution, 20 levels

u, v, T, q, Ps, Tg

$$360 \times 180 \times 20 = 1.3 \times 10^6 \times 4 \text{ variabl} = 5 \times 10^6$$

- **observation** :  $10^4 \sim 10^5$  **non-uniform distribution**  
 $\pm 3 \text{ hour window}$

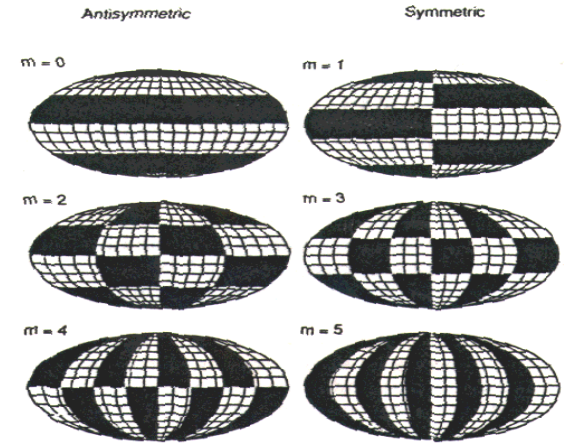
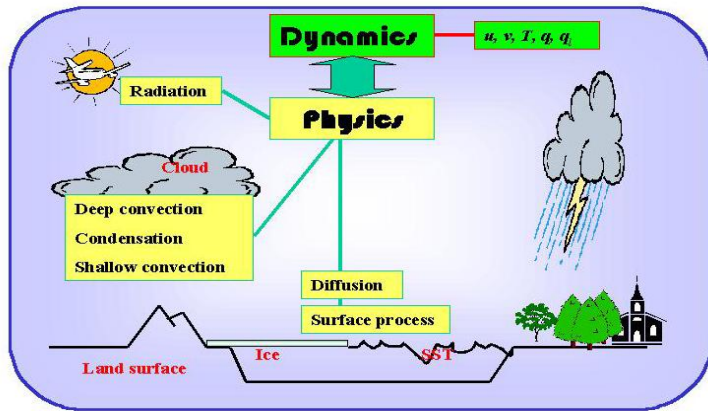
- Data assimilation cycle
  - 1) data checking
  - 2) objective analysis
  - 3) Initialization: dynamical adjustment
  - 4) short-range fcst for first guess



# Model

- Dynamics : Speed
- Physics : Predictability

# Step3: Integration



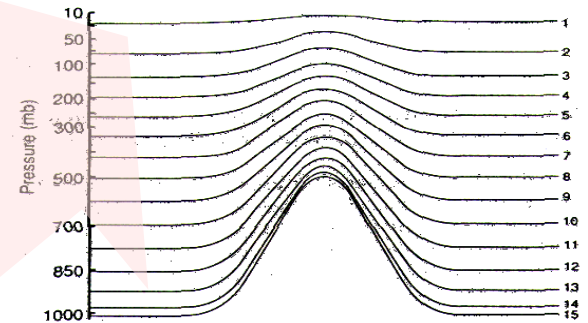
$$\frac{\partial u}{\partial t} = -u \frac{\partial u}{\partial x} - v \frac{\partial u}{\partial y} - \omega \frac{\partial u}{\partial p} - \frac{\partial \Phi}{\partial x} + fv + F_x$$

$$\frac{\partial v}{\partial t} = -u \frac{\partial v}{\partial x} - v \frac{\partial v}{\partial y} - \omega \frac{\partial v}{\partial p} - \frac{\partial \Phi}{\partial y} - fu + F_y$$

$$\frac{\partial \Phi}{\partial t} = -\frac{RT}{p}$$

$$\frac{\partial T}{\partial t} = -u \frac{\partial T}{\partial x} - v \frac{\partial T}{\partial y} + \omega \left( \frac{\kappa T}{p} - \frac{\partial T}{\partial p} \right) + \frac{\dot{H}}{c_p}$$

$$\frac{\partial \omega}{\partial p} = -\left( \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right)$$



15-level sigma-coordinate model

# Dynamics : model frame

Finite difference method (FDM) :

Spectral method (SPM) :

Finite element method (FEM) :

Ex)  $\frac{\partial \phi}{\partial t} = -c \frac{\partial \phi}{\partial x}$ ; advection eq.

1) FDM (유한 차분법)

$$\frac{\Delta \phi}{\Delta t} = \frac{\phi_2 - \phi_1}{t_2 - t_1}$$

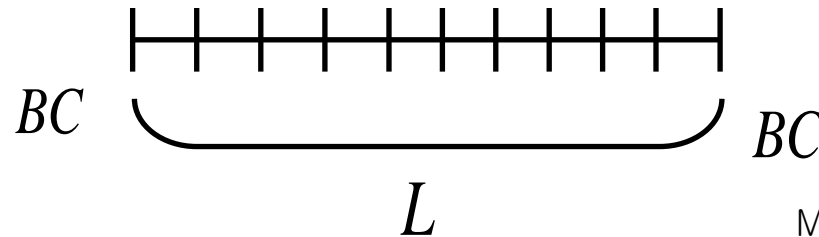
2) Spectral Method (분광법)

- Determine basis function to get  $H(\phi(x))$

- Expand  $\phi$  in terms of a time series

$e_m(x)$  (basis funct),  $m = m_1 \dots m_n \dots \rightarrow$  infinite

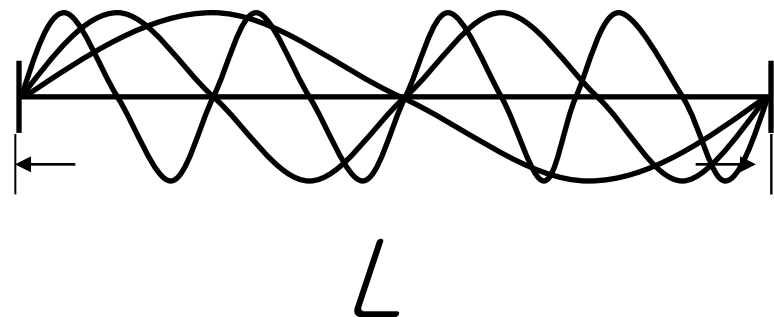
$$\Rightarrow \phi(x, t) = \sum_{m=m_1}^{m_2} \phi_m(t) e_m(x)$$



No variation between grid points

M 개의 grids

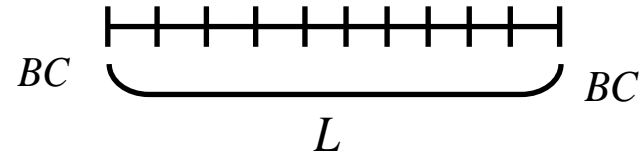
M 개의 function



\* Resolution Increases  $\left\{ \begin{array}{l} \Delta x \rightarrow \text{decreases} \\ m \rightarrow \text{increases} \end{array} \right.$  19

## Dynamics : model frame (continued)

Ex)  $\frac{\partial u}{\partial t} = -cu_x$  ; advection eq.



No variation between grid points

### Integration scheme

{	Explicit :	$\frac{u^{n+1} - u^n}{\Delta t} = F(u^n)$	: forward
		$\frac{u^{n+1} - u^{n-1}}{\Delta t} = F(u^n)$	: leap-frog $\Rightarrow$ conditionally stable
	Implicit :	$\frac{u^{n+1} - u^n}{\Delta t} = F(u^{n+1})$	: fully implicit
		$\frac{u^{n+1} - u^n}{\Delta t} = F\left(\frac{u^n + u^{n+1}}{2}\right)$	: Crank-Nicholson <span style="color: red;">absolutely stable</span>

\* Summary

a)  $\frac{u^{n+1} - u^{n-1}}{2\Delta t} = F(u^n)$  : leap-frog **good for hyperbolic**  
**unstable for parabolic**

b)  $\frac{u^{n+1} - u^n}{\Delta t} = F(u^n)$  : Euler-forward **good for diffusion**  
**unstable for hyperbolic**

c)  $\frac{u^{n+1} - u^n}{\Delta t} = F\left(\frac{u^n + u^{n+1}}{2}\right)$  : **Crank-Nicholson**

d)  $\frac{u^{n+1} - u^n}{\Delta t} = F(u^{n+1})$  : **Fully implicit, backward**

e)  $\frac{u^* - u^n}{\Delta t} = F(u^n)$  :  $\frac{u^{n+1} - u^n}{\Delta t} = F(u^*)$  : **Euler-backward (Matzuno)**

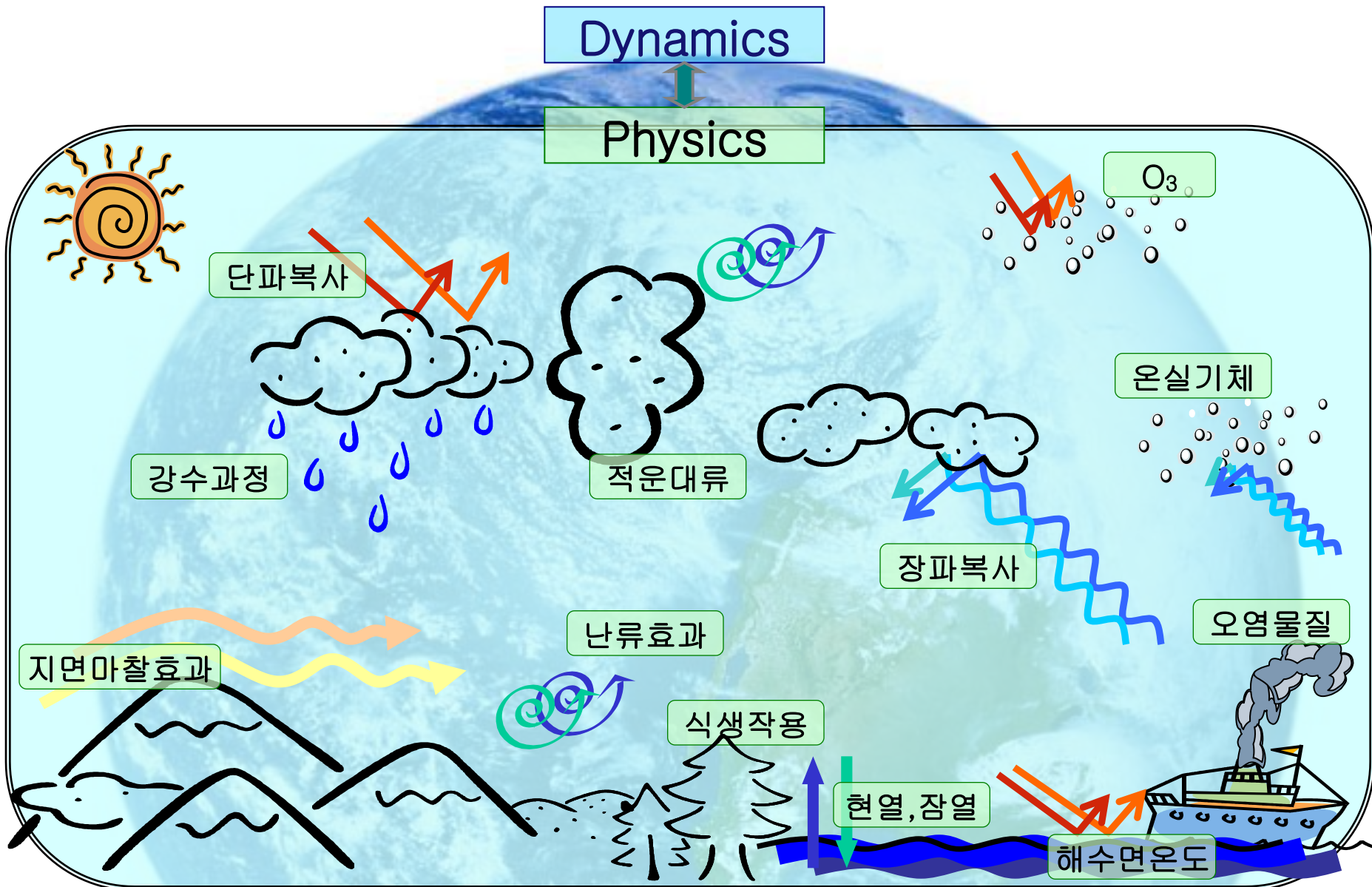
f)  $\frac{u^{n+\frac{1}{2}*} - u^n}{\Delta t/2} = F(u^n)$  :  $\frac{u^{n+\frac{1}{2}**} - u^n}{\Delta t/2} = F\left(u^{n+\frac{1}{2}*}\right)$

$\frac{u^{n+1} - u^n}{\Delta t} = \frac{1}{6} \left[ F(u^n) + 2F\left(u^{n+\frac{1}{2}*}\right) + 2F\left(u^{n+\frac{1}{2}**}\right) + F(u^{n+1*}) \right]$  : **RK(Runge-Kuta)-4th order**

g)  $\frac{u^{n+1} - u^{n-1}}{2\Delta t} = F_1(u^n) + F_2\left(\frac{u^{n+1} - u^{n-1}}{2}\right)$  : **Semi-Implicit**

h)  $\frac{u^* - u^n}{\Delta t} = F_1(u^n)$ ;  $\frac{u^{n+1} - u^*}{\Delta t} = F_2(u^*)$  : **Fractional steps**

# Numerical modeling



# Classification of models

- Dynamic frame

<b>Hydrostatic</b>	<b>Non-hydrostatic</b>
Large-scale	Small-scale (heavy rainfall, complex mountain)

- Scale

<b>Global</b>	<b>Regional</b>
10 km – 100 km	1 km–10 km

- Purpose

<b>FORECAST</b>	<b>Forcing → RESPONSE</b>
<b>NWP</b> : upto 2 weeks	<b>GCM</b> (General circulation model)



## Weather Research and Forecasting (WRF) Model

ADMINISTRATION

CALENDAR

GROUP HOME PAGES

DIRECTORY OF  
DEVELOPERS

USERS

DOCUMENTATION/  
PRESENTATIONS

FORUM

REAL-TIME WRF  
FORECASTS

ANNOUNCEMENTS/  
UP COMING EVENTS

**NEW!** **Announcing WRF V2.0 Release** (May 18, 2004)

### What is in WRF V2.0?

- Advanced Research WRF (ARW) dynamical core: Eulerian mass coordinate
- One-way and two-way nesting
- A few new physics options, including Noah LSM, RUC LSM, YSU PBL, and Grell-Devenyi ensemble cumulus scheme
- ESMF time manager
- Enhanced I/O options
- New SI V2.0
- WRF 3DVAR V2.0

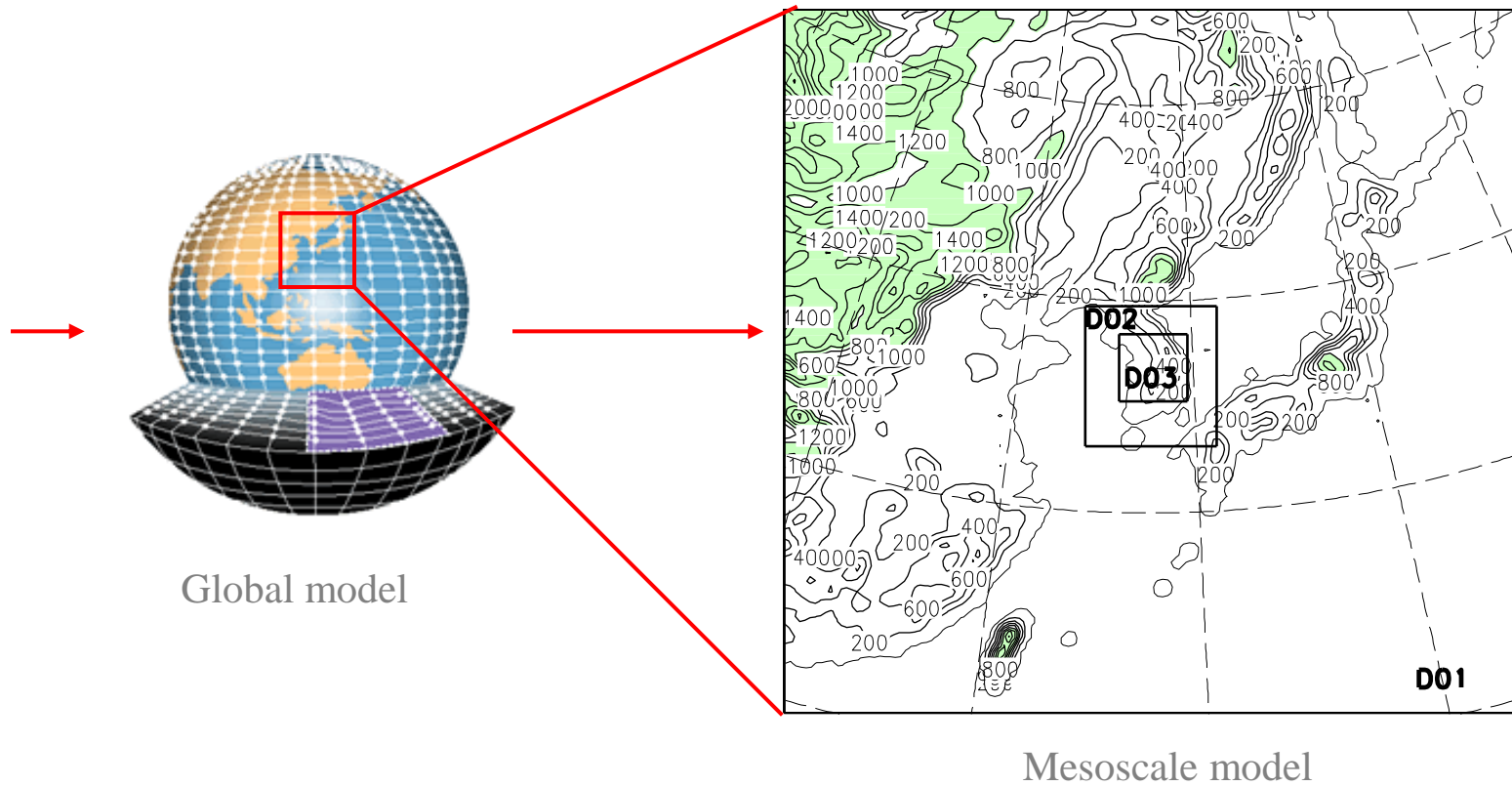
### How do you get WRF V2.0?

Simply select the [USERS](#) button to the left. Information on the User's page is updated for the new release. You may download WRF software and test datasets from the site.

# WRF Model

The Weather Research and Forecasting Model (WRF)

⚙ Mesoscale grid model



## Dynamics : model frame (continued)

In real NWP model (<http://wrf-model.org>)

### Governing equations in flux form

Vertical coordinate:  $\eta$ -coord.  $\eta = (p - p_t) / \mu$  where  $\mu = p_s - p_t$

Flux-form variables:  $\vec{V} = \mu \vec{w} = (U, V, W)$ ,  $\Omega = \mu \dot{\eta}$ ,  $\Theta = \mu \theta$ ,

#### 1) Temporal Discretization

A time-split integration scheme is used.

##### For low-frequency modes:

Runge-Kutta time integration scheme

$$\Phi^* = \Phi^t + \frac{\Delta t}{3} RK(\Phi^t)$$

$$\Phi^{**} = \Phi^t + \frac{\Delta t}{2} RK(\Phi^*)$$

$$\Phi^{t+\Delta t} = \Phi^t + \Delta t RK(\Phi^{**})$$

##### For high-frequency acoustic modes:

Integrated over smaller time steps to maintain numerical instability

$$\partial_t U + (\nabla \cdot \vec{V}u) - \partial_x (p \partial_\eta \phi) + \partial_\eta (p \partial_x \phi) = F_U$$

$$\partial_t V + (\nabla \cdot \vec{V}v) - \partial_y (p \partial_\eta \phi) + \partial_\eta (p \partial_y \phi) = F_V$$

$$\partial_t W + (\nabla \cdot \vec{V}w) - g(\partial_\eta p - \mu) = F_W$$

$$\partial_t \Theta + (\nabla \cdot \vec{V}\theta) = 0$$

$$\partial_t \mu + \nabla \cdot \vec{V} = 0$$

$$\partial_t \phi + \mu^{-1} [(\vec{V} \cdot \nabla \phi) - gW] = 0$$

## Dynamics : model frame (continued)

### In real NWP model

### Governing equations in flux form

Vertical coordinate:  $\eta$ -coord.  $\eta = (p - p_t) / \mu$  where  $\mu = p_s - p_t$

Flux-form variables:  $\vec{V} = \mu \vec{w} = (U, V, W)$ ,  $\Omega = \mu \dot{\eta}$ ,  $\Theta = \mu \theta$ ,

$$\partial_t U + (\nabla \cdot \vec{V} u) - \partial_x (p \partial_\eta \phi) + \partial_\eta (p \partial_x \phi) = F_U$$

$$\partial_t V + (\nabla \cdot \vec{V} v) - \partial_y (p \partial_\eta \phi) + \partial_\eta (p \partial_y \phi) = F_V$$

$$\partial_t W + (\nabla \cdot \vec{V} w) - g(\partial_\eta p - \mu) = F_W$$

$$\partial_t \Theta + (\nabla \cdot \vec{V} \theta) = 0$$

$$\partial_t \mu + \nabla \cdot \vec{V} = 0$$

$$\partial_t \phi + \mu^{-1} [(\vec{V} \cdot \nabla \phi) - gW] = 0$$

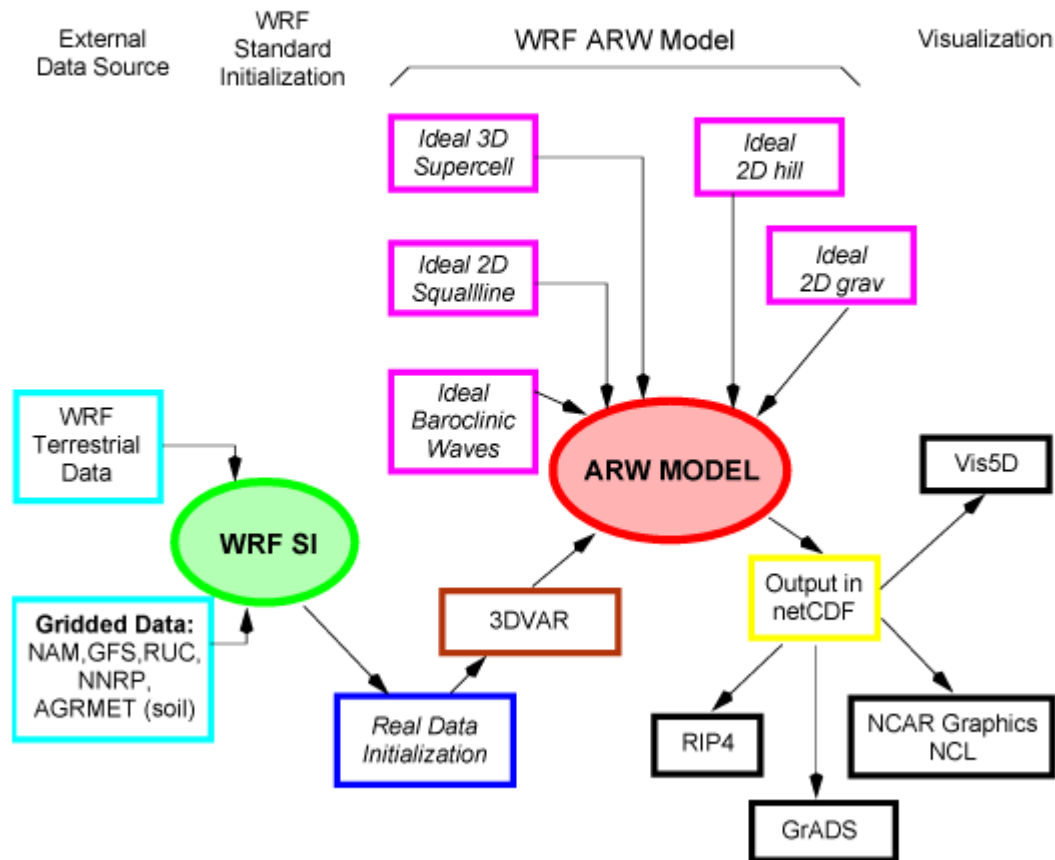
### 2) Spatial Discretization

Ex) Horizontal momentum

$$\partial_t U'' + \overline{\mu^{t^*}} \overline{\alpha^{t^*}} \delta_x p''^t + (\overline{\mu^{t^*}} \delta_x \overline{p}) \overline{\alpha^{t^*}} + \overline{(\alpha / \alpha_d)}^x \left[ \overline{\mu^{t^*}} \delta_x \overline{\phi^{t^*}}^\eta + (\delta_x \overline{\phi^{t^*}}^\eta) (\delta_\eta \overline{p^{t^*}}^x - \overline{\mu^{t^*}}^x)^t \right] = RK_U^{t_27}$$

# WRF Model Process

WRF ARW Modeling System Flow Chart (for WRFV2)



# Cloud and precipitation



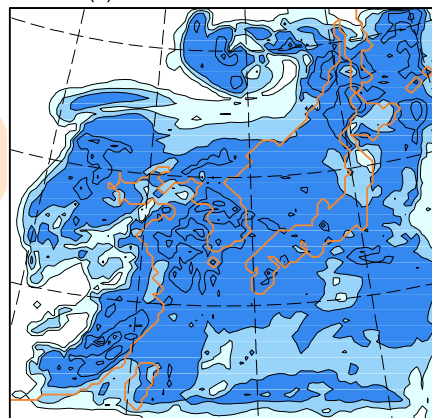
# WSMMP (WRF-Single-Moment- MicroPhysics)

## Hong, Dudhia and Chen (2004)

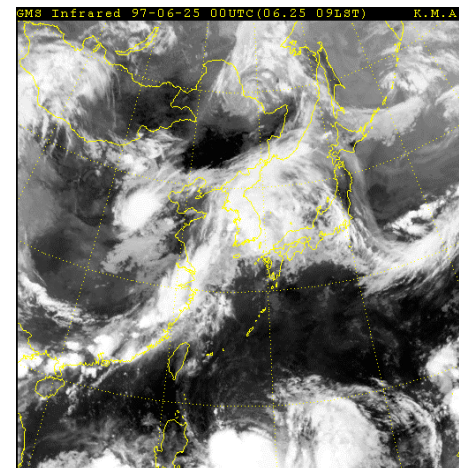
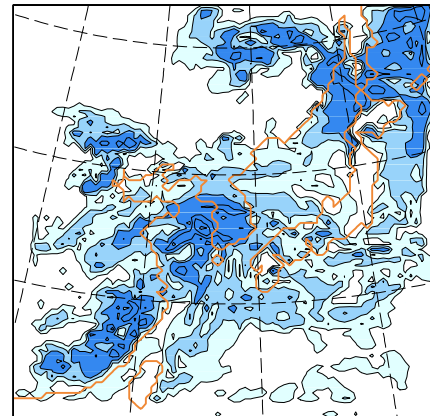
Major modifications suggested by Hong et al. (2004)

	(Rutledge and Hobbs, 1983)	(Hong et al, 2003)
Number concentration of cloud ice	$N_I (m^{-3}) = 10^{-2} \exp[0.6(T_0 - T)]$	$N_I = c(r q_I)^d$
Ice nuclei number	$N_I (m^{-3}) = 10^{-2} \exp[0.6(T_0 - T)]$	$N_{I0} = 10^3 \exp[0.1(T_0 - T)]$
Intercept parameter for snow	$N_{0S} = 2 \cdot 10^7 m^{-4}$	$N_{0S} (m^{-4}) = 2 \cdot 10^6 \exp\{0.12(T_0 - T)\}$

old



new



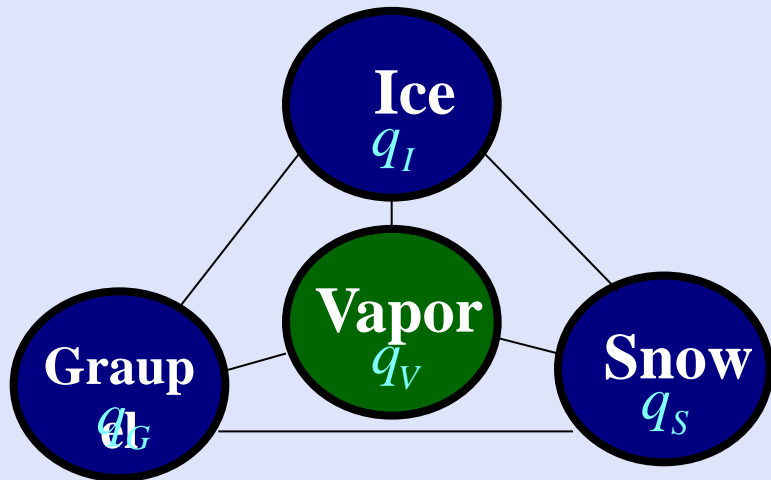
23 –25 June 1997  
Heavy Rainfall Case  
(Vertically integrated cloud ice)

# Weather Research and Forecasting (WRF) Double-Moment 6-class (WDM6) Microphysics scheme

Lim and Hong (2010)

**Cold rain processes :**

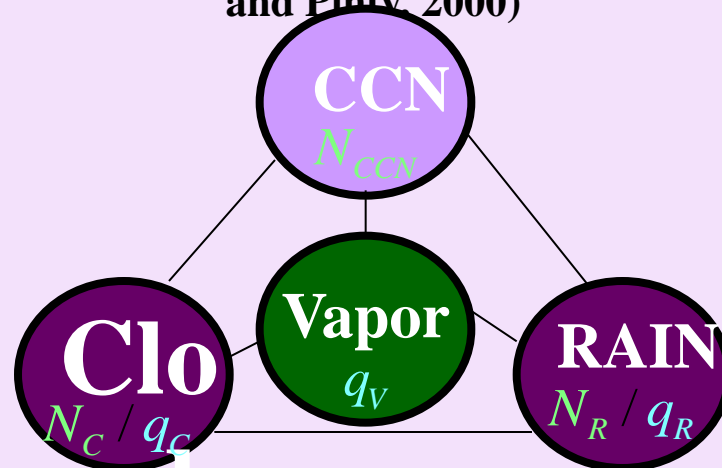
(Hong et al.,2004; Hong and Lim 2006)



q for **4** hydrometeors will be predicted  
(**S**ingle **M**oment)

**Warm rain processes :**

(Khairoutdinov and Kogan, 2000; Cohardt and Pinty, 2000)



N,q for **2** hydrometeors will be predicted  
(**D**ouble **M**oment)

WDM6

warm rain processes

- follows the double-moment processes in Lim and Hong

```
do k = kts, kte
  do i = its, ite
    supsat = max(q(i,k),qmin)-qs(i,k,1)
    satdt = supsat/dtclld
```

```
praut: auto conversion rate from cloud to rain [CP 17]
(C->R)
```

```
lencon = 2.7e-2*den(i,k)*qci(i,k,1)*(1.e20/16.*rslopec2(i,k)
  *rslopec2(i,k)-0.4)
lenconcr = max(1,2*lencon, qcrmin)
if(avedia(i,k,1).gt.di15) then
  taucon = 3.7/den(i,k)/qci(i,k,1)/(0.5e6*rslopec(i,k)-7.5)
  praut(i,k) = lencon/taucon
  praut(i,k) = min(max(praut(i,k),0.),qci(i,k,1)/dtclld)
```

```
nraut: auto conversion rate from cloud to rain [CP 18 & 19]
(NC->NR)
```

```
nraut(i,k) = 3.5e9*den(i,k)*praut(i,k)
if(qrs(i,k,1).gt.lenconcr)
  nraut(i,k) = ncr(i,k,3)/qrs(i,k,1)*praut(i,k)
  nraut(i,k) = min(nraut(i,k),ncr(i,k,2)/dtclld)
endif
```

```
pracw: accretion of cloud water by rain [CP 22 & 23]
(C->R)
```

```
nracw: accretion of cloud water by rain
(NC->)
```

```
if(qrs(i,k,1).ge.lenconcr) then
  if(avedia(i,k,2).ge.di100) then
    nracw(i,k) = min(ncrk1*ncr(i,k,2)*ncr(i,k,3)*(rslopec3(i,k)
      + 24.*rslope3(i,k,1)),ncr(i,k,2)/dtclld)
    pracw(i,k) = min(pi/6.*(denr/den(i,k))*ncrk1*ncr(i,k,2)
      *ncr(i,k,3)*rslopec3(i,k)*(2.*rslopec3(i,k)
      + 24.*rslope3(i,k,1)),qci(i,k,1)/dtclld)
  else
    nracw(i,k) = min(ncrk2*ncr(i,k,2)*ncr(i,k,3)*(2.*rslopec3(i,k)
      *rslopec3(i,k)+5040.*rslope3(i,k,1)
      *rslope3(i,k,1)),ncr(i,k,2)/dtclld)
    pracw(i,k) = min(pi/6.*(denr/den(i,k))*ncrk2*ncr(i,k,2)
      *ncr(i,k,3)*rslopec3(i,k)*(6.*rslopec3(i,k)
      *rslopec3(i,k)+5040.*rslope3(i,k,1)*rslope3(i,k,1))
      ,qci(i,k,1)/dtclld)
```

```
endif
endif
```

\*\* Warm rain processes (Hong and Lim 2010)

\*Auto conversion from cloud to rain [C → R]

$$\text{Praut} [\text{kgkg}^{-1}\text{s}^{-1}] = L / \tau \quad L = 2.7 \times 10^{-2} \rho_a q_c \left( \frac{10^{20}}{16 \lambda_c^4} - 0.4 \right)$$

$$\tau = 3.7 \frac{1}{\rho_a q_c} \left( \frac{0.5 \times 10^6}{\lambda_c} - 7.5 \right)^{-1}$$

$$\text{Nraut} [\text{m}^{-3}\text{s}^{-1}] = 3.5 \times 10^9 \frac{\rho_a L}{\tau}$$

\*Accretion of cloud water by rain [C → R]

$$D_R \geq 100 \mu\text{m}$$

$$\text{Pracw} [\text{kgkg}^{-1}\text{s}^{-1}] = \frac{\pi \rho_w}{6 \rho_a} K_1 \frac{N_C N_R}{\lambda_c^3} \left\{ \frac{2}{\lambda_c^3} + \frac{24}{\lambda_R^3} \right\}$$

$$\text{Nracw} [\text{m}^{-3}\text{s}^{-1}] = -K_1 N_C N_R \left\{ \frac{1}{\lambda_c^3} + \frac{24}{\lambda_R^3} \right\}$$

$$D_R < 100 \mu\text{m}$$

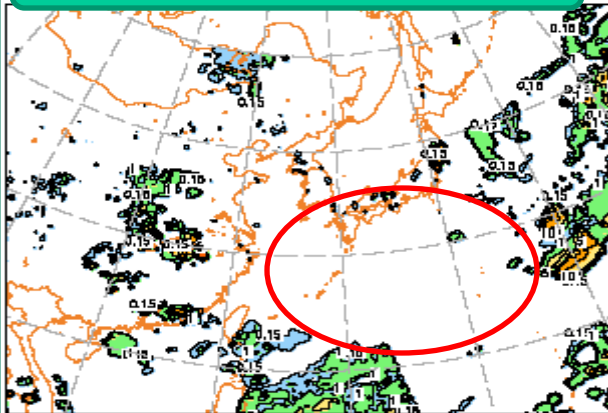
$$\text{Pracw} [\text{kgkg}^{-1}\text{s}^{-1}] = \frac{\pi \rho_w}{6 \rho_a} K_2 \frac{N_C N_R}{\lambda_c^3} \left\{ \frac{6}{\lambda_c^6} + \frac{5040}{\lambda_R^6} \right\}$$

$$\text{Nracw} [\text{m}^{-3}\text{s}^{-1}] = -K_2 N_C N_R \left\{ \frac{2}{\lambda_c^6} + \frac{5040}{\lambda_R^6} \right\}$$

# Precipitation in Spring

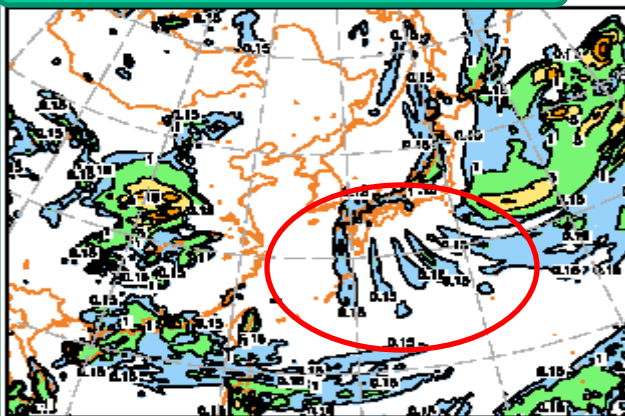
-last 6 hour accumulated precipitation

**TMPA : rainfall estimates**

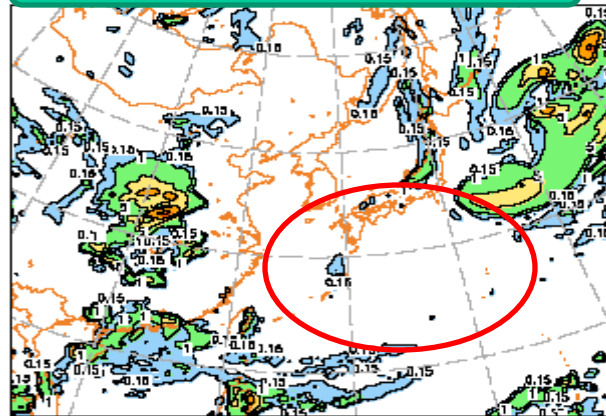


WDM6 (WRF-double-moment 6-class cloud scheme ) that predicts CCN, Nc, Nr will be released in WRF in April 2009 (Version 3.1)

**WSM6 : Hong and Lim (2006)**

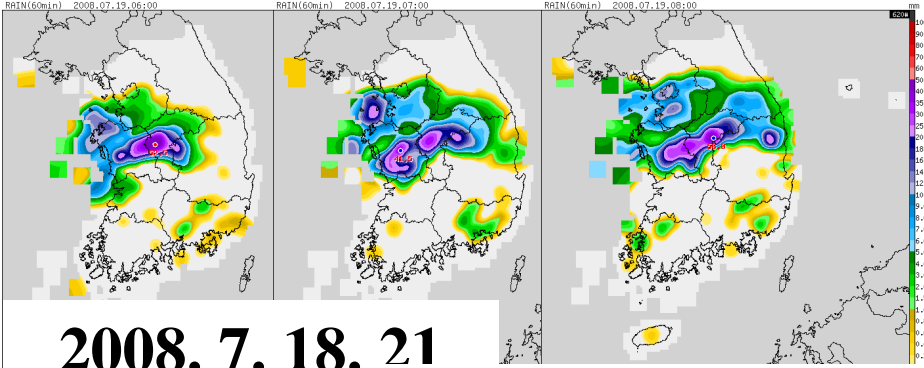


**WDM6 : Lim and Hong (2009)**



# Precipitation (hourly)

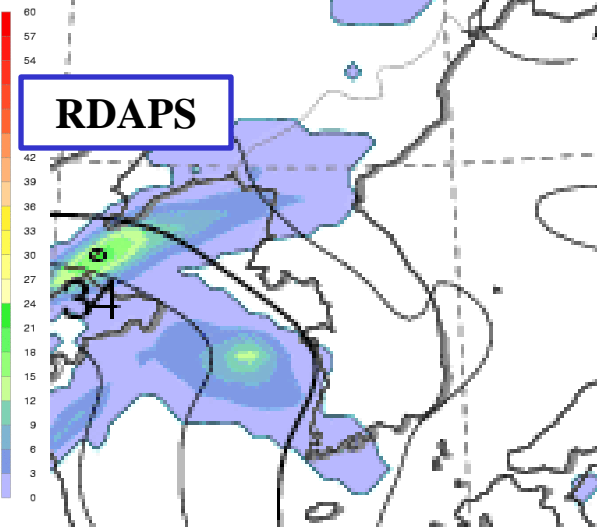
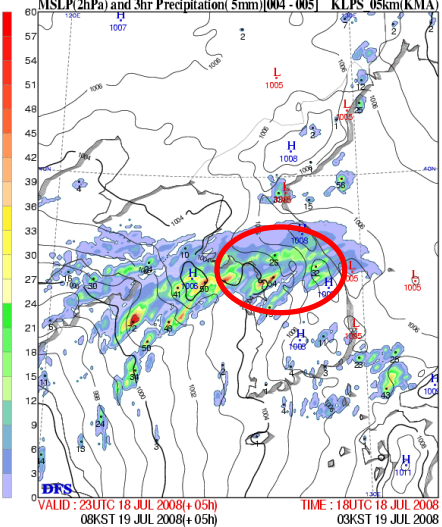
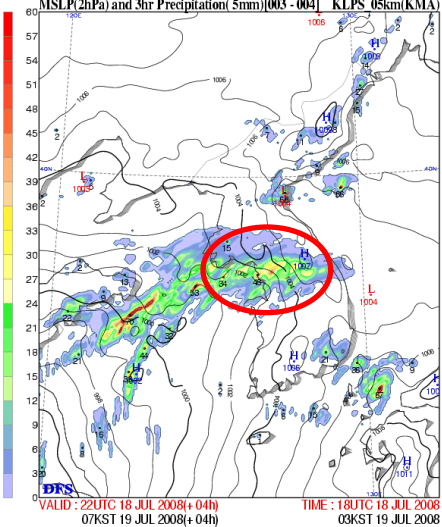
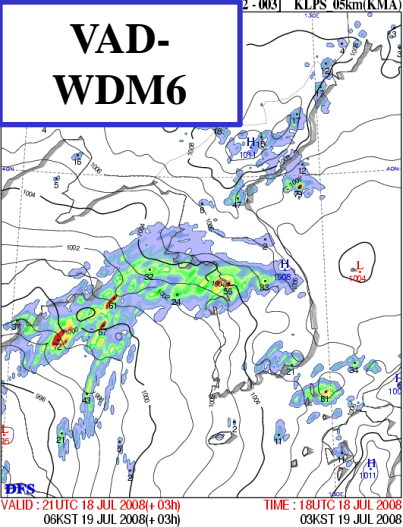
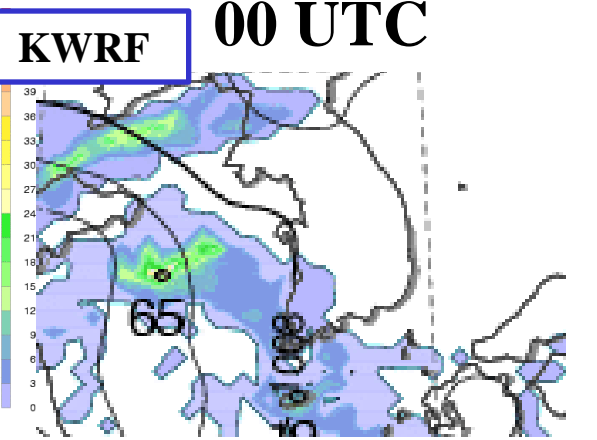
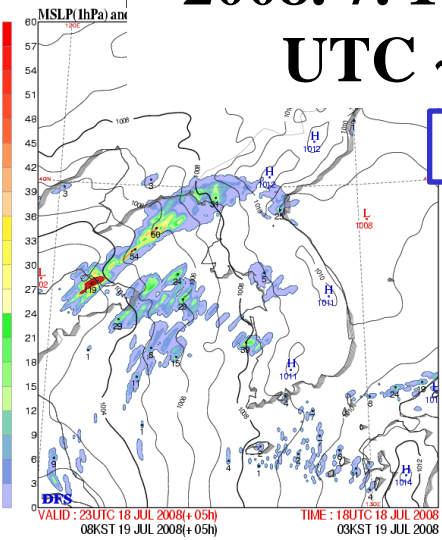
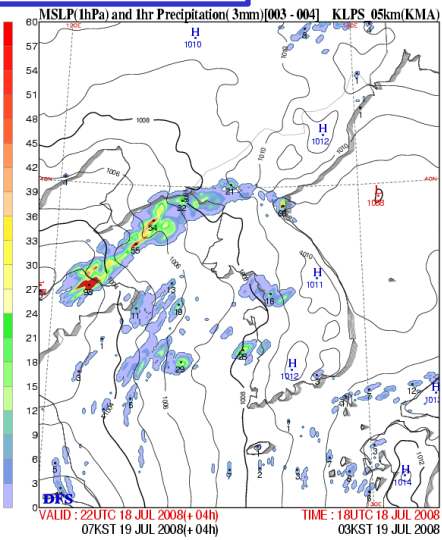
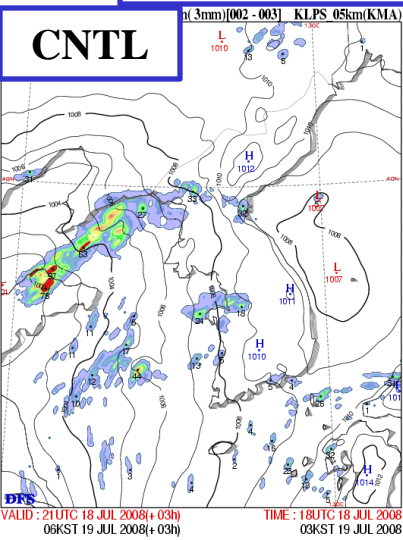
**2008. 7. 18. 18 UTC (3-5 fc)**



**2008. 7. 18. 21**

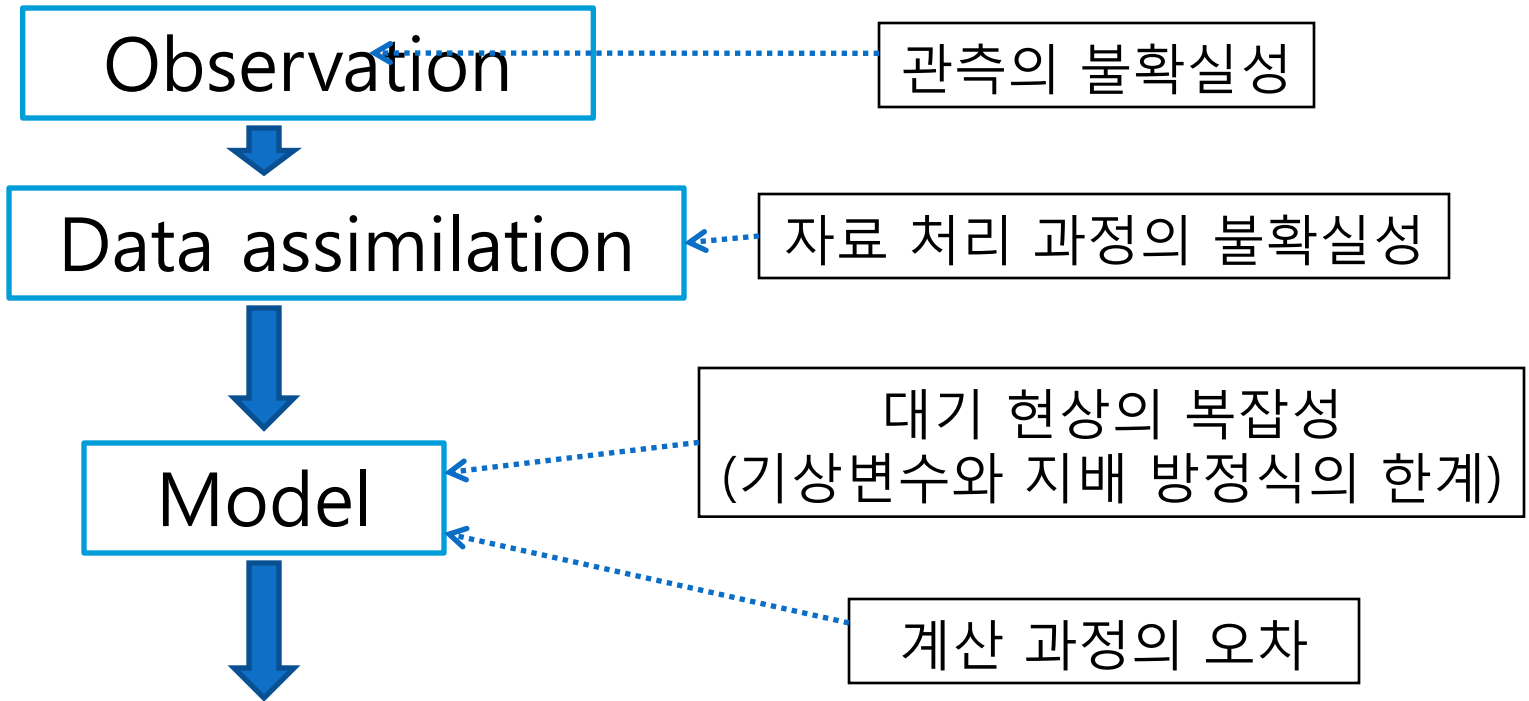
**UTC ~ Valid : 2008. 7. 19.**

**00 UTC**



# Predictabilities

# Uncertainties





# Chaos theory (Lorenz)



**Charney (1951) : Uncertainties in initial condition and model**

**Lorenz (1962,1963) : Unstable nature of atmosphere**

**Purpose : NWP is better than statistical forecast**

**Tool : 4 K memory computer**

**Model : 12 variables (heating and dissipation forcing)**

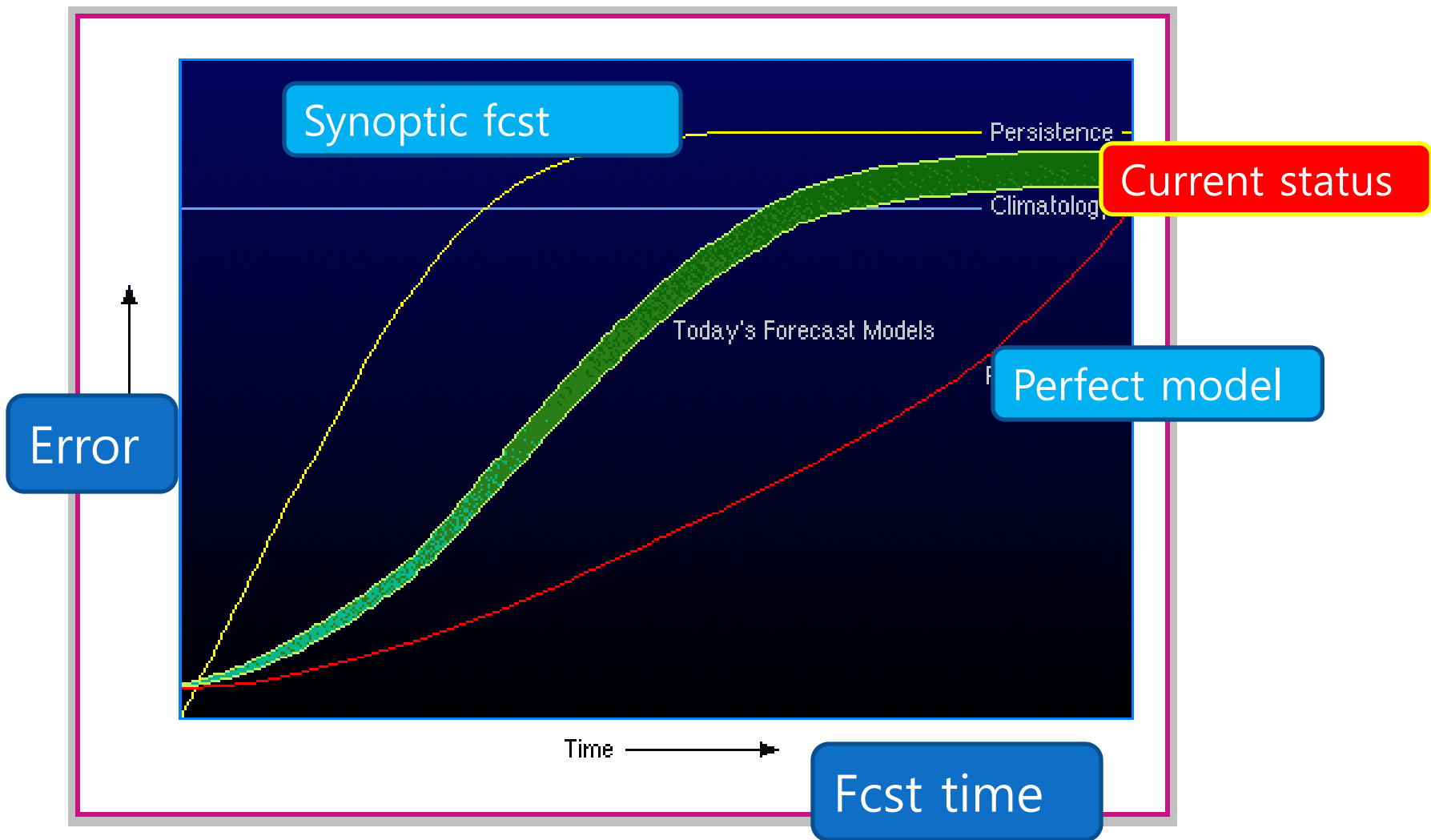
**Results : differences -> non-periodicity**

**Initial condition (3 decimal point) : different after 2 month**

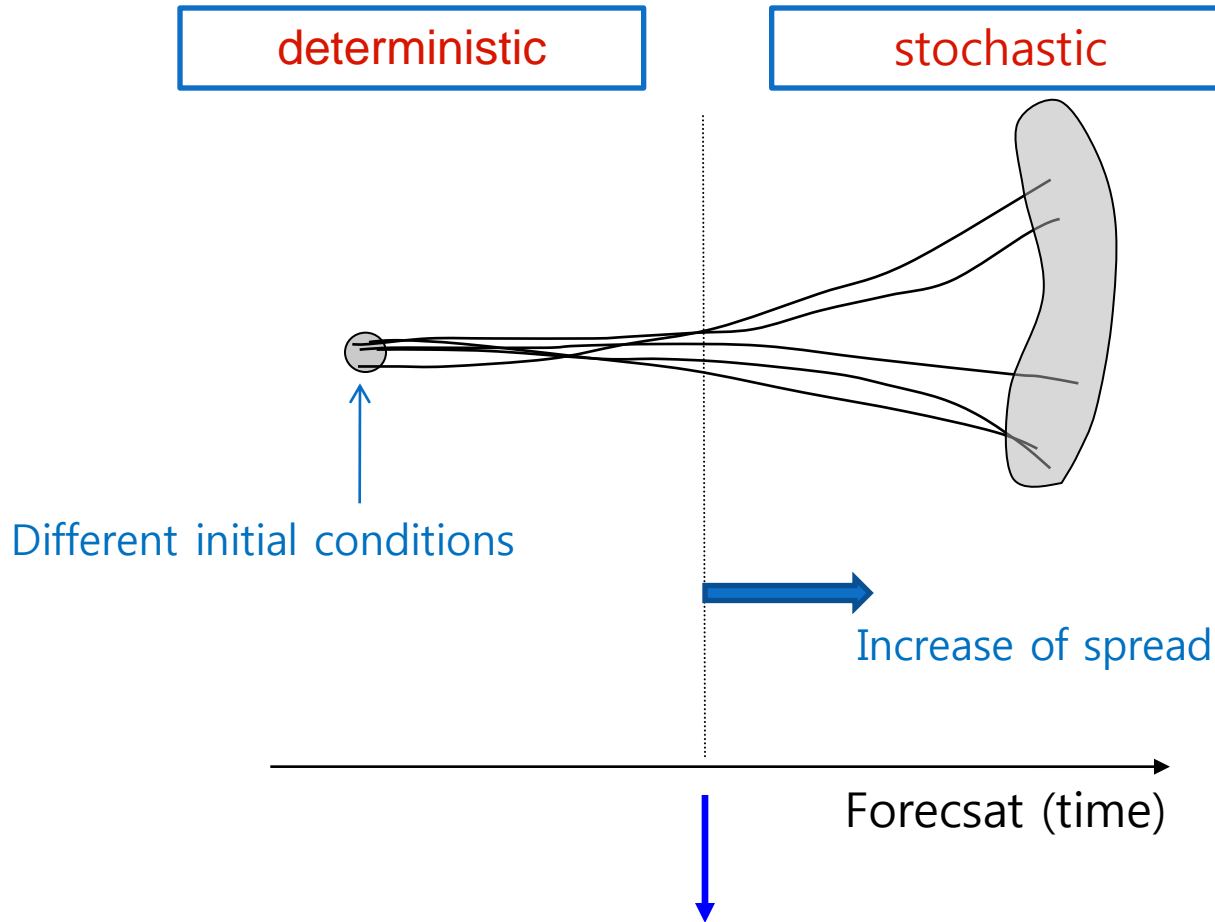
**Round-off error -> cause of non-periodicity**

**Chaos theory– two weeks**

# Predictability

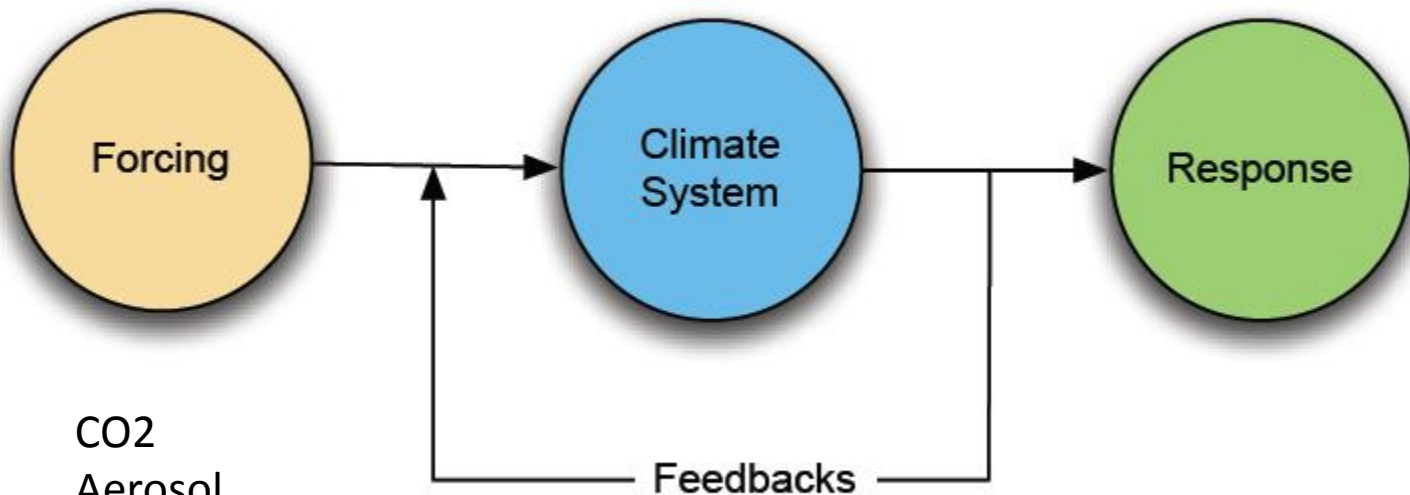


# Ensemble forecasts



**Large-scale flows : a few days, mesoscale : a few hours**

# Climate system sensitivity



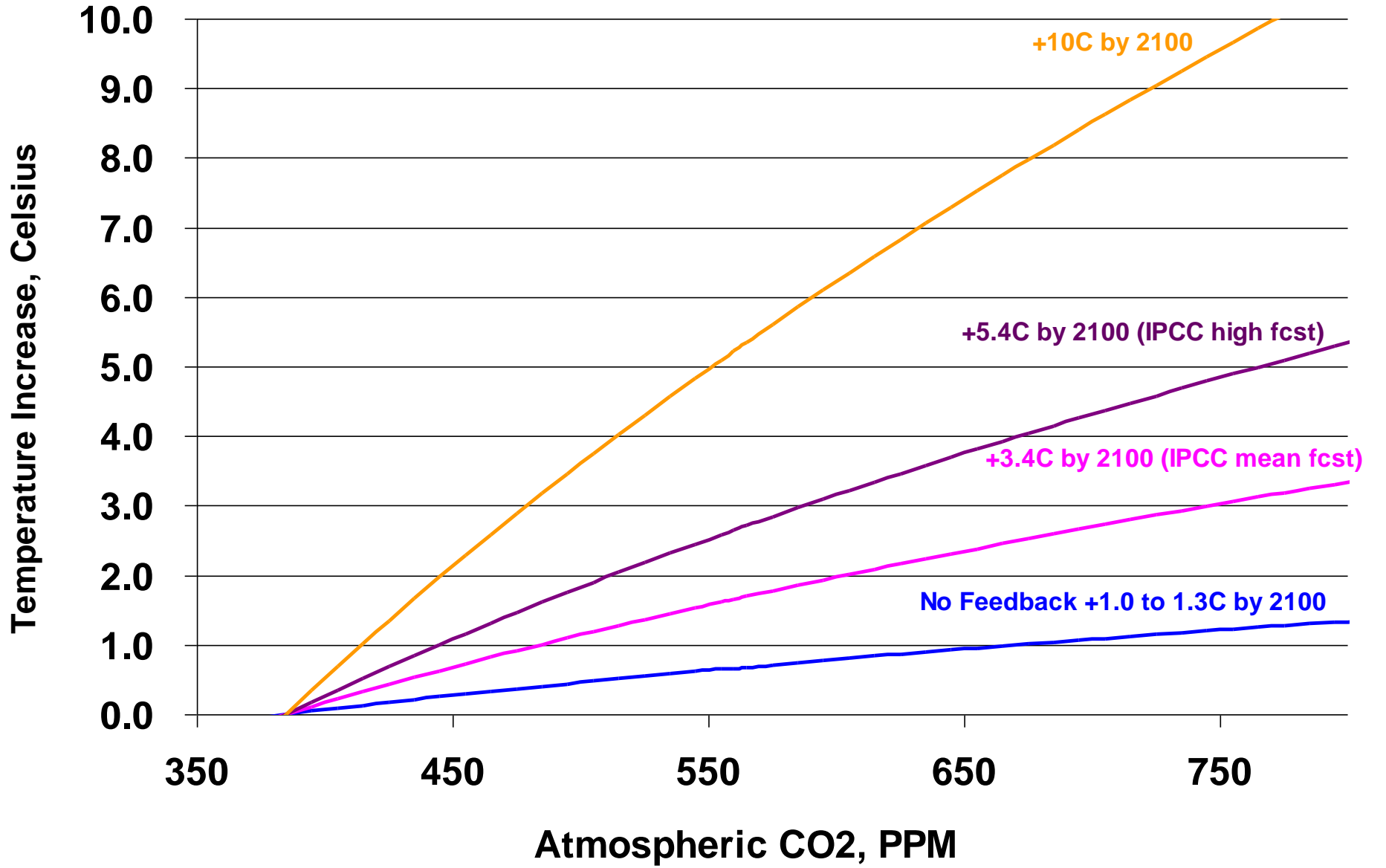
CO2  
Aerosol  
Volcanic

Water vapor feedback  
Ice-albedo feedback  
Vegetation feedbacks  
Cloud (radiative) feedback  
(Great debate!, Mostly still uncertain)

...

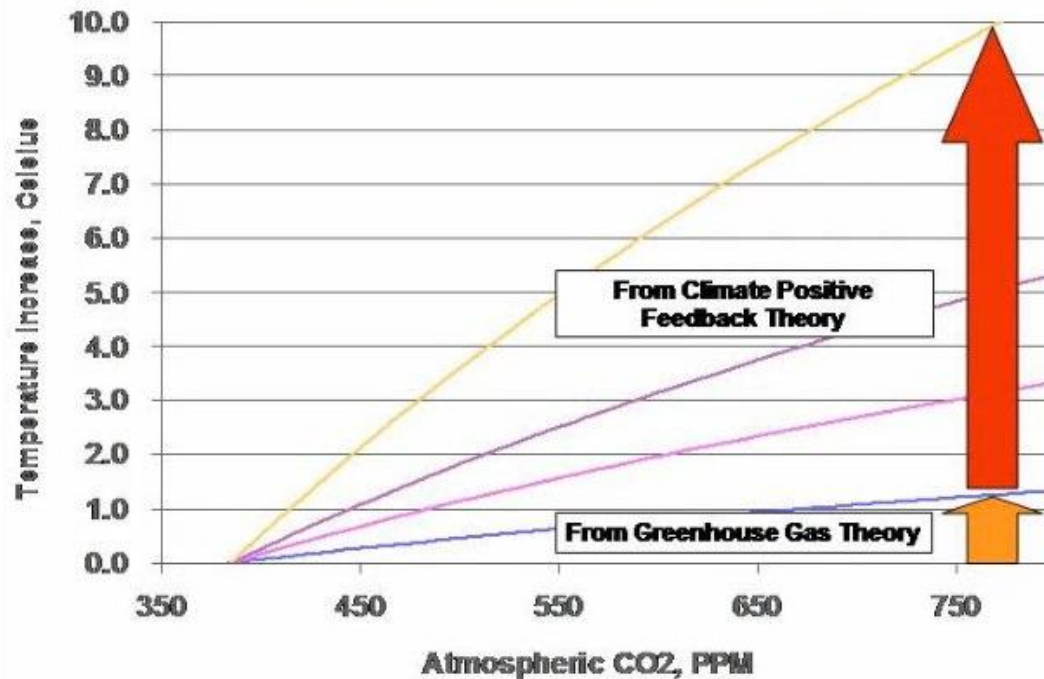
# Temperature Projections From CO2

## IPCC A2 (no Abatement) Case



# Positive feedback !!!

Catastrophic Global Warming Theory Based on Two Chained Theories



Also, uncertainties are pronounced ....

**END ~ ~ ~**